Syllabus

Advanced Calculus, AB

Course Overview

Calculus is the mathematics of change. It is used to solve complex problems that are continuously evolving and would otherwise be unsolvable with only algebra and geometry. This online advanced placement course is designed to prepare students to become deep mathematical thinkers. You will explore the calculus concepts of limits, differentiation, and integration and apply those concepts in meaningful ways.

The course is split into two semesters. The first semester focuses on the concepts of functions, limits, and differentiation and their applications. The second semester builds off the first semester to focus on integrations. It will cover topics such as the definite and indefinite integral and their applications, inverse function, and techniques for integrating.

Course Goals

By the end of the course the student will be able to:

- Work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal, and understand the connections among these representations.
- Understand the meaning of the derivative in terms of a rate of change and local linear approximation and use derivatives to solve a variety of problems.
- Understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and use integrals to solve a variety of problems.
- Understand the relationship between the derivative and the definite integral as expressed in both parts of the fundamental theorem of calculus.
- Communicate mathematics both orally and in well-written sentences and explain solutions to problems.
- Model a written description of a physical situation with a function, a differential equation, or an integral.
- Use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Math Skills

Successful completion of four years of high school mathematics provides the mathematical skills students need for Advanced Calculus. Typically, this includes Algebra 1, Algebra 2, Geometry, and Precalculus (including elements of Trigonometry) or four years of integrated high school mathematics.

General Skills

To participate in this course, you should be able to do the following:

- Complete basic operations with word processing software, such as Microsoft Word or Google Docs.
- Understand the basics of spreadsheet software, such as Microsoft Excel or Google Spreadsheets, but having prior computing experience is not necessary.
- Perform online research using various search engines and library databases.
- Communicate through email and participate in discussion boards.

For a complete list of general skills that are required for participation in online courses, refer to the Prerequisites section of the Plato Student Orientation document, found at the beginning of this course.

Credit Value

Advanced Calculus is a 1.0-credit course. It includes semesters A and B.

Course Materials

- notebook
- graphing calculator, recommend TI-83 or equivalent
- computer with Internet connection and speakers or headphones
- Microsoft Word or equivalent

Teaching Strategies

Structure

This online course is organized into units and lessons. Each lesson incorporates multiple learning activities designed to develop, apply, and assess specific learning objectives. (See Course Outline, below.)

Concept Development Activities
In order to generate skills for lifelong learning and to employ the most appropriate learning approach for each topic, twenty-five percent of the lessons will use student-driven, constructivist approaches for concept development. The remaining lessons will employ direct instruction approaches. In either case, students will take full advantage of the online learning environment, linking to rich online, multimedia, and interactive resources. Developing critical 21st Century skills is an important secondary goal of this course.

Application
Application and inquiry will be an integrated part of the lessons, requiring higher-level cognitive work. Students will submit written work online for review, comment, and grading.

Discussions/Group Work
Students will also have the opportunity to engage in online (asynchronous) discussions during this course. Discussion topics provide the chance to dig deeper into specific calculus and STEM concepts and applications.

Throughout AP Calculus, students will also have the opportunity to work synchronously in groups. Group work will allow students a chance to communicate orally about the mathematical concepts in the course and to provide explanations on how they derived solutions to the problems presented in the course. Much of the oral discussions will emphasize real world application/explanation of mathematical concepts.

For example, in Semester A, Unit 2, students will participate in a discussion activity about the mathematical meaning of the word “limit” and how that compares to the more commonly used meaning that they normally hear and use. They will communicate both orally and in writing for this and other discussions in the course.

Pedagogy

- **STEM:** This course will make special effort to integrate the study of calculus in a broader STEM perspective. In today’s global economy, employers are looking for skilled workers who are innovative problem solvers and critical thinkers. Such skills encompass what STEM education is. It fuses science, technology, engineering, and mathematics into one cohesive discipline. STEM education is designed to open students’ minds to the exciting and fulfilling opportunities in STEM-related careers.

  Many guidelines have been integrated into the course design and lessons that embody the goals of a STEM education. Lessons will use real-world scenarios that that will illustrate to students how math, science, engineering, and technology are applied in professional and everyday lives. Students will leverage technologies such as graphing

calculators, spreadsheets, and the Internet to help them solve difficult and complex problems. Students will also be given culminating activities that will require them to solve open-ended, complex problems.

- **Constructivism:** Twenty-five percent of the course will use student-driven, constructivist approaches in order to generate skills for lifelong learning. The remaining portion of the course will be direct instruction on topics of importance to the subject matter.

- **Context:** All too often, concepts are brought to the abstract algebraic level before students have a good understanding of what the concept is. Lessons in this course will attempt to explore the concept in an easy to understand, real-world manner. That will help learners grasp the concept within a familiar context so that they can make a clear connection to the abstract.

- **Reasoning and Sense-Making**: It's important to allow students to develop their reasoning and sense-making skills. Students need to be able to think about mathematics in a way that is meaningful to them. In these lessons, students will be asked to generate logical conclusions and make meaningful connections to both the known and unknown.

- **Practice**: As important as it is for students to be able to understand the concepts in their own way, they still have to be able to perform and master the algebraic manipulation required to complete this class. Therefore, practice will be integrated into the lessons.

- **Technology:** Technology is changing at an exponential rate. It is important that students are familiar with technology and how to use it appropriately. This course will make use of current technologies, such as the Internet, graphing calculators, and spreadsheets to help students solidify their understanding of the concepts that are presented in this course.

    - **Graphing calculators** – students will need a graphing calculator in this course to explore, experiment, and investigate calculus concepts. The course will provide an embedded graphing tool to use within lessons, but the student will be expected to have a handheld version as well. Graphing calculator activities require the student to explore what happens when you change points on the graph and require the student to draw conclusions based on those changes. Specific graphing calculator activities are detailed in the unit outline below. Students can either purchase the calculators for themselves or they will be provided for them by the school or teacher.

Student Evaluation

Multiple evaluation tools will be used to assess understanding at all appropriate cognitive levels and to reflect AP assessment methodology:

- **Lesson-Level Mastery Tests:** Each lesson will be accompanied by an AP-style multiple-choice mastery test to assess mastery of the basic lesson concepts.

- **Self-Assessment Lesson Activities:** Especially useful in constructivist-leaning lessons, self-assessment activities will provide sample responses against which learners can assess their own learning.
- **Teacher-Graded Lesson Activities:** These lesson activities will require teacher assessment, employing AP-style objective rubrics. Students will be provided with the rubrics for each assignment.
- **Unit-Level Posttests:** Each unit will have a multiple-choice assessment to confirm that all the material within the unit has been retained and can be applied in a larger context than a single-lesson format.
- **Unit-Level Culminating Activities:** Learners will have the chance to apply their knowledge of the concepts across the lessons within a unit. Most of the units will include this teacher-graded activity for evaluation of higher order thinking skills.
- **End-of-Semester Tests:** At the end of each of the two semesters, learners will take a multiple-choice test to assess mastery of concepts and provide additional practice for a long-form exam like the AP exam.

Course Outline

This course will be structured in two 18-week semesters. Semester A will be divided into four units and semester B will be divided into five units, as follows.

Units

Semester A

Functions/Prerequisites for Calculus
Limits
Derivatives
Applications of Derivatives

Semester B

The Definite Integral
Applications of Integration
Inverse Functions
Techniques of Integration
Further Applications of Integration

Readings

This course will employ the following textbook as a resource for deep research and learning:

Edwards, C. H, and Penney, D.E. *Calculus*, *6th Edition*. Upper Saddle River, NJ: Prentice Hall, Inc. 2002 Schedule, Topics, and Objectives.

AP Topic Outline

Detailed objectives for all course lessons are documented by unit in the course outline below. This scope and sequence is designed to introduce basic concepts and applications first, then to spiral back to more sophisticated understandings and applications. In the process, the AP topic outline is fully met and built up in a logical and natural way. The following guide concisely maps the AP topic outline to the detailed course outline below:

I. **Functions, Graphs, and Limits**
The following AP topics are covered in semester A, units 1 and 2. Higher-level functions are investigated in semester B, unit 3:

- analysis of graphs
- limits of functions (including one-sided limits)
- asymptotic and unbounded behavior
- continuity as a property of functions

II. **Derivatives**
The following AP topics are covered in semester A, units 3 and 4. Derivatives for higher-level functions are investigated in semester B, unit 3:

- concept of the derivative
- derivative at a point
- derivative as a function
- second derivatives
- applications of derivatives
- computation of derivatives

III. Integrals
The following AP topics are covered in semester B, units 1, 2, 3, 4, and 5:

- interpretations and properties of definite integrals
- fundamental theorem of calculus
- techniques of antidifferentiation
- applications of antidifferentiation
- numerical approximations to definite integrals

Semester A

Unit 1 - Functions/Prerequisites for Calculus (2-3 Weeks, 10 Lessons)

Functions – Define relationships called functions and learn about the parts of a function.

- *define functions*
- *explore the meaning of function inputs*
- *explore the meaning of function outputs*
- *understand the function rule*
- *understand how the different parts of a function are related*

Describing Functions with Equations, Tables, and Graphs – Learn about how equations, tables, and graphs can all represent the same function.

- *explore different ways to represent a function*

Defining a Function with Its Rule – Learn how to determine whether a relation represents a function.

- *review the meaning of a relation*
- *define what makes a relation a function*
- *determine whether a relation is a function*

Identifying Graphs from Their Equations – Identify the graph of an equation.

- *understand, identify, and calculate slope*
- *understand, identify, and calculate y-intercepts*
- *graph an equation in slope-intercept form*

Parallel Lines and Their Slopes – Identify parallel lines and their slopes.

- *calculate the slope of a line*
- *recognize parallel lines*
- *identify the slopes of parallel lines*

Perpendicular Lines and Their Slopes – Identify perpendicular lines and their slopes.

- *calculate the slope of a line*
- *recognize parallel lines*
- *identify the slopes of parallel lines*

Equations of Parallel or Perpendicular Lines – Identify equations of parallel or perpendicular lines.

- *determine when the equations of two lines represent parallel lines*
- *determine when the equations of two lines represent perpendicular lines*

- *determine when the equations of two lines represent neither parallel nor perpendicular lines*

Review: Graphs – Graph linear equations and identify parallel and perpendicular lines.

- *Identify the slope and points for a line on the coordinate plane*
- *determine if two lines are parallel*
- *determine if two lines are perpendicular*

Solving Systems of Linear Equations – Use the graphing method to solve a system of two linear equations.

- *graph two linear equations to find the solution to the system of equations*
- *determine the number of solutions that exist for a system of two linear equations*

Review: Linear Systems – Solve systems of linear equations or inequalities using a variety of methods.

- *solve systems of linear equations and inequalities by graphing*
- *solve systems of linear equations using substitution*
- *solve systems of linear equations using addition*
- *solve systems of linear equations using matrices*

Solving Problems with Linear Systems – Use systems of linear equations or inequalities to solve real-world problems.

- *model real-world problems using systems of linear equations or inequalities*
- *solve systems of linear equations and inequalities using a variety of methods*

Types of Functions - Categorize and describe functions.

- *explore categories of functions while noting the format of the equation as well as its graph: linear, polynomial, power, rational, algebraic, inverse, exponential, logarithmic*
- *explore piecewise functions and a specific example of a piecewise function, the absolute value function*
- *explore descriptors of functions such as even, odd, increasing, decreasing, symmetry, polynomial, degree, quadratic, and cubic*
- *review trigonometric angle measurement*
- *calculate missing measurements given lengths and angles of a triangle*
- *graph trigonometric functions*
- *use trigonometric identities to solve and simplify trigonometric expressions*
- *review trigonometric identities, functions and their graphs.*
- *review trigonometric angle measurement*
- *calculate missing measurements given lengths and angles of a triangle*

Graphing Solution Sets of Associated Inequalities – Solve and graph the solution sets of quadratic inequalities.

- *solve quadratic inequalities*
- *solve inequalities that require unions of solution sets*
- *graph solution sets for quadratic inequalities*

Equations of Ellipses and Hyperbolas – Recognize the equations of ellipses and hyperbolas and graph them.

- *identify equations of ellipses and hyperbolas*
- *calculate features of ellipses and hyperbolas such as centers and axes*

Graphing Linear Inequalities in 1 Variable – Learn to graph the solution sets for single-variable inequalities.

- *solve and graph linear inequalities using interval notation*
- *solve and graph linear inequalities using a number line*

Translations and Transformations – Apply translations and transformations to function graphs and their equations.

- *translate, rotate, and reflect function graphs*
- *use function notation to represent function transformations*
- *rewrite functions to represent a transformation*

Functional Values – Compute functional values by translating and transforming a function.

- *Find points on a transformed graph based on function notation and a point on the original graph*
- *Use function transformations to solve real-world problems*

Composite Functions – Learn to find composite functions and compute the points on the resulting function graph.

- *Use composition to combine two functions*
- *Find points that satisfy a composite function*
- *Determine the simpler functions that make up a composite function*

Unit 2 – Limits (3-4 Weeks, 8 Lessons)

Tangent Lines and Velocity - Examine the tangent problem and relate it to instantaneous velocity.

- *Understand a tangent line graphically and how it relates to a rate of change*
- *view a tangent line approaching from the left and right sides*
- *relate a tangent problem to instantaneous velocity*

- *Instantaneous Velocity Activity*
  - calculate the slope and instantaneous velocity using data from a table or a function without the use of a calculator
  - given a function and table of ranges, calculate the average velocity for each interval by hand
  - explain the methods used to find answers
  - determine the instantaneous velocity at a given point
- *graphing calculator activity*
  - Use the graphing tool (or your personal calculator) to see what happens to the secant when you reduce the interval between points P and Q:
  - Enter the function f(x) = 0.25*(x^2).
  - Enter 2 (the x-coordinate of P) as the value of the tangent at x. Enter 4 (the x-coordinate of Q) as the value of the secant at x. Then click New Function to generate the graph.
  - Reduce the value of the x-coordinate of Q so it is closer to the value of the x-coordinate of P. Note how the graph changes.
  - What can you conclude from this activity?

What Are Limits? - Examine limits using both a numerical and graphical approach.

- *understand the concept of limits as seen graphed or stated as a function*
- *use the definition of a limit to find the limit of a function*
- *evaluate limits from the left side, right side, and both sides*

Laws of Limits? - Calculate limits using the limit laws.

- *review and apply laws for constant, sum, product, quotient and exponential functions*

Continuous and Discontinuous Limits - Evaluate limits that are continuous and discontinuous.

- *examine different types of discontinuities (removable, jump) on a graph*
- *identify functions that have infinite discontinuities*
- *use the theorems of continuity to show a function is continuous*

Evaluation of Limits - Evaluate limits algebraically.

- *identify limits that are indeterminate at some value "c"*
- *develop strategies for evaluating limits algebraically*

The Squeeze Theorem - Evaluate trigonometric limits.

- *review the squeeze theorem graphically and algebraically*
- *apply the squeeze theorem to trigonometric limits*
- *graphing calculator activity*
    - Use the graphing calculator to graph the functions *f*, *g*, and *h* (based on the notation of the squeeze theorem).

The Intermediate Value Theorem - Apply the Intermediate Value Theorem to find at least one solution.

- *use the intermediate value theorem to find a solution to a given problem*
- *lesson activity*
    - *ensure that the hypotheses of the intermediate value theorem are satisfied in a given situation*
    - *apply the intermediate value theorem once the hypotheses are verified*
- *use the bisection method to estimate f(x)= 0 on an interval*
- *graphing calculator activity*
    - Roots and Intervals Using a Graphing Calculator
        - Use a graphing calculator to prove that the equation cos $x = x^2$ has at least one root. Note the interval in which the root lies. Apply the Bisection Method to find a more accurate interval of length 0.01. *Hint: Use f(x) = cos x – x2.*
        - Use a graphing calculator to prove that the equation x4 + x3 - 3 = 0 has at least one root. Note the interval in which the root lies. Apply the Bisection Method to find the root, rounding up to 2 decimal places.

The Formal Definition of a Limit - Evaluate a limit using the precise definition of a limit.

- *examine the precise definition of a limit graphically*
- *apply the precise definition of a limit to a function on a continuous interval*

Unit 3 – Derivatives (4-5 Weeks, 11 Lessons)

Definition of the Derivative - Explore the relationship between the derivative and a tangent line.

- *analyze the geometric (or graphic) interpretation of the derivative*
- *find the derivative of a function at a number "a" using the definition of a derivative*
- *use the definition of a derivative to find the slope of a tangent line or curve at a given point*
- *find the equation of a tangent line at a given point*

- *graphing calculator activity*
    - In this secant tool, you will see the graph of the function cosh(x) and two sliders below the graph. Note that the top slider changes the point $[f(x+h),(x+h)]$ while the bottom slider changes the point$[f(x),x]$. Experiment with the two sliders to see how the slope of the secant line changes, and then answer the following questions.
        - Now that you've experimented with the sliders, use the bottom slider to fix the value of *x* to *x* = 1. Use the top slider to change the values of *x* + *h* to those given in the following table. Record the slope of the secant for each case.
        - What happens to the secant as *h* approaches 0?
        - Can you guess the slope of the tangent at *x* + *h* = 1
    - For the function f(x) = 2x2 + x, use the difference quotient $\frac{f(x)-f(a)}{x-a}$ to find the slope of the secant joining the points at x = 1 and a = 2.
    - For the function f(x) = 2x2 + x, complete the table with the values of the difference quotient to estimate the slope at the point x = 1.
        - Based on what you have learned in this lesson activity, what limit formula can be used to find the tangent line of a function at a specific point?

Instantaneous Rate of Change - Use the derivative to represent an instantaneous rate of change.

- *calculate the rate of change from function*
- *interpret the rate of change within the context of a problem*
- *approximate the rate of change from graphs and tables*

Graphing the Derivative - Use the derivative as a function.

- *explore the relationship between the graph of a function and its derivative*
- *match the graph of a function with the graph of its derivative function*
- *understand the various notations for the derivative*
- *graphing calculator activity*
    - First play around with the Calculus Grapher tool to explore the relationship between the graphs of a function and its derivative. You can create a function graph by dragging the line below and above the x-axis. As you create a graph, its derivative graph is automatically created. Here are some things you can do to explore different graphs in the tool:
        - Create graphs of different shapes using the buttons on the right.
        - Combine different shapes and observe the corresponding
        - Derivatives.

        - Use the slider to stretch your graphs along the x-axis.
    - For every change you make in the graph, watch carefully how the derivative changes as you distort the function.
    - Activate the Grid and Cursor from the panel on the right and compare the values of the function and its derivative. Once you've familiarized yourself with the tool, go to the Calculus Grapher section of your Lesson Activities document, where you'll find a list of instructions to perform with the Calculus Grapher and questions to answer.

Differentiability and Continuity - Determine when a function can fail to be differentiable.

- *understand the difference between differentiability and continuity*
- *identify instances when the derivative does not exist (corner, discontinuity, vertical tangent)*

The Formal Definition of the Derivative - Differentiate a function using the basic rules of differentiation.

- *develop the rules for finding derivatives by using the formal definition of the derivative*

Differentiation Rules - Learn and apply rules for differentiating functions.

- *find the derivative using the constant function, power function and constant multiple rules*
- *find the derivative of a sum or difference function using the rules of differentiation*
- *find the derivative using the product rule*
- *find the derivative using the quotient rule*
- *find the derivative using the general power rule*

Higher Order Derivatives - Explore higher order derivatives.

- *find higher-order derivatives (i.e. second, third, etc.)*
- *identify the curves on a graph that represent the original function as well as the first, second and third derivatives*
- *use higher order derivatives to determine rates of change (velocity, acceleration, jerk)*

Differentiation of Trigonometric Functions - Differentiate trigonometric functions.

- *develop rules for finding derivatives of trigonometric functions by using the definition of the derivative*
- *review limits of trigonometric functions*
- *find the derivative of a trigonometric function*
- *explore the use of simplification using the trigonometric identities prior to finding the derivative of a function involving multiple trigonometric functions*

The Chain Rule - Differentiate using the chain rule.

- *demonstrate differentiation on composite functions by using the chain rule*
- *demonstrate the use the power rule combined with the chain rule to find the derivative of composite power function*

Differentiation of Implicit Functions - Perform implicit differentiation.

- *describe how and when implicit differentiation is used*
- *perform implicit differentiation*
- *use technology to explore the graphs of an equation and its derivative*

Differentiation in the Real World - Apply differentiation techniques in rates of change problems.

- *explain the difference between the average rate of change and the instantaneous rate of change*
- *setup and calculate problems to find the rate of change*

Unit 4 – Application of Derivatives (4-5 Weeks, 10 Lessons)

Linear Approximations - Use linear approximation and graphs to solve problems requiring linearization.

- *define how "differentials" can be used as independent and dependent variables*
- *perform linearization of a function*
- *investigate approximation error and accuracy*

Fermat's Theorem and Maximum and Minimum Values - Identify extrema on an interval from a graph.

- *find maximum and minimum values on open and closed intervals (identify as absolute or local)*
- *describe the difference between absolute and local maximum (and minimum) values*
- *identify extreme values on continuous and non-continuous intervals*
- *use Fermat's theorem to find local maximum and minimum values of a function*
- *identify critical numbers of a function on a closed interval*

The Mean Value Theorem - Explore the mean value theorem.

- *explore a graphical representation of the mean value theorem*
- *use Rolle's theorem to show the proof of the mean value theorem*
- *use the mean value theorem to find a number "c" that satisfies the conclusion*
- *understand the mean value theorem from a geometrical perspective*
- *explore the three hypotheses of Rolle's theorem*
- *use Rolle's theorem to find a number where Rolle's theorem is satisfied*

The First Derivative Test - Examine the first derivative test and use it to aid in graphing.

- *explore the use of the first derivative test graphically*
- *use the first derivative test to identify the interval on which a function is increasing and decreasing*
- *apply the first derivative test to identify local maximum or local minimum values at a critical number "c"*
- *graph a function while using the results of the first derivative test*

The Second Derivative Test - Examine the second derivative test and use it to aid in graphing.

- *explore the use of the second derivative test graphically*
- *use the second derivative test to identify intervals on which a function is concave upward or downward*
- *use the second derivative test to identify any points of inflection*
- *use the second derivative test to identify local maximum and minimum values*
- *graph a function while using the results of the second derivative test*

Limits at Infinity and Asymptote - Identify limits at infinity.

- *determine limits at infinity*
- *determine horizontal asymptotes of a graph or function (if they exist)*
- *use the precise definition of a limit to find the corresponding number N*
- *graphing calculator activity*

    - Using a graphing calculator, find the vertical asymptote for the function. Write down the steps you took to reach your answer.
    - Using a graphing calculator, find the vertical asymptote for the function. Write down the steps you took to reach your answer.

Curve Sketching - Apply the techniques from limits and derivatives to curve sketching.

- *demonstrate how the use function basics as well as first and second derivative tests and asymptotes to aid in sketching a curve*
- *demonstrate how calculators or technology can aid in curve sketching*

Comparing Related Rates - Calculate related rates problems.

- *use the derivative to find the solution to problems that involve the rate of change of one quantity in terms of the rate of change of another quantity*

Maximum and Minimum Applications - Perform differentiation in order to solve applied problems.

- *In this lesson, students will:*
    - *solve applied maximum and minimum problems in the areas of physics, business and economics*
    - *solve optimization problems using provided equations or verbal descriptions of situations*

Newton's Method - Explore Newton's method graphically and algebraically.

- *explore Newton's method graphically*
- *perform the necessary calculations involved in Newton's method to find the approximate value*

Semester B

Unit 1 – The Definite Integral (4-5 Weeks, 10 Lessons)

The Antiderivative - Explore the concept of the antiderivative by using the derivative.

- *explore the definition of the antiderivative*
- *understand the use of "C" when finding the antiderivative*
- *create the graph of an antiderivative using a given function*
- *find the antiderivative in its general form and with given specific conditions*

- *graphing calculator activity*
    - Assume a function, $F(x) = \sin x + C$, is an antiderivative of a function *f(x)*. Using a graphing calculator, substitute different values for C and plot each function. Do you notice any similarity in the graphs of these functions? Note your observations.

Area Under a Curve - Explore the area problem graphically and by using summation notation.

- *estimate the area below a curve using by splitting the region into rectangles (2, 4, 8, etc.)*
- *estimate distance traveled by using the velocities at various times*
- *state the differences in estimating the area below a curve using the left endpoint, right endpoint and midpoint for curves that are increasing and decreasing*
- *find the area below a curve by splitting the region into "n" rectangles while using the left and right endpoints as "n" approaches infinity*
- *Area Under the Curve Activity*
    - *interpret and make use of sigma notation and limit notation to solve for the area under a curve*
    - *make use of smaller intervals of estimation to find better estimations for the area under a curve*
    - *make use of limits to express the exact area under a curve as the Reimann sum with the number of subdivisions tending toward infinity*

The Definite Integral - Examine the definition of the definite integral.

- *perform integration by using the definition of the definite integral*
- *understand how formulas for sums are used in finding the definite integral*
- *compare area found using sum to definition of definite integral*
- *explore how the area is affected by curves that are above the x-axis, below the x-axis, and both above and below the x-axis*

Properties of the Definite Integral - Demonstrate the use of the properties of the definite integral.

- *equate switching endpoints of a definite integral with the negative of a definite integral*
- *show that the area from a to a on a definite integral is zero*
- *find the value of definite integrals involving a constant, sum and difference*

Comparison Properties of the Definite Integral - Explore comparison properties of the definite integral.

- *review the comparison properties of the definite integral graphically*
- *use the comparison properties to insure a plausible answer for a definite integral*
- *lesson activity*
  - *plot the graphs of two given functions*
  - *compare the functions and write down any observations in sentence form*

The Fundamental Theorem of Calculus - Explore the fundamental theorem of calculus graphically and by using the definite integral.

- *explain the fundamental theorem of calculus graphically*
- *examine the proof of the fundamental theorem of calculus (parts 1 and 2)*
- *use the fundamental theorem of calculus to find the value of a definite integral*
- *graphing calculator activity*
  - Plot the functions in the table for the given values of *a* and *x*. Observe the graph of each function, its integral, and the antiderivative of the integral. Write your observations in the table. The first one has been done for you.
  - Explore the geometric aspect of the theorem. Use the same functions you plotted in the earlier graph. Change the values of *a* and *b*, observe the graphs, and write your observations in the table.

Indefinite Integrals - Use the tables of indefinite integrals to perform integration.

- *explain the difference between a definite and indefinite integral*
- *find the definite and indefinite integrals of constants, simple functions, and trigonometric functions using the table of indefinite integrals*

Applications for the Definite Integral - Explore the applications of definite integrals in terms of distance and displacement.

- *find the integral of a velocity function to determine displacement*
- *use the net change theorem to examine displacement vs. distance*
- *use the net change theorem in other practical application problems*

The Substitution Rule - Perform integration by using the substitution rule.

- *identify when the strategy of substitution is used to find definite and indefinite integrals*
- *evaluate integrals using substitution*

Symmetric Functions and Integration - Perform integration of symmetric functions.

- *examine why a function must be continuous to find a definite integral*
- *apply the properties of symmetric functions to find during integration*

Unit 2 – Applications of Integration (2-3 Weeks, 6 Lessons)

Area Between Two Curves - Determine the area between two curves.

- *graphically estimate the area between two curves*
- *use "n" rectangles to find the area between two curves*
- *use the formula for area to find the area between two curves with given boundaries*
- *use the formula for area to find the area between two intersecting curves*
- *examine area problems where areas need to be split into more than one integral*
- *identify and calculate areas where the functions are identified as x = f(y)*
- *graphing calculator activity.*
    - Using a graphing calculator, plot the functions $y = \sin x + 5$ and $y = \cos x + 3$ on a coordinate plane. Shade the region under the curve $y = \sin x + 5$ over the interval [2, 5] and label it A1. On a second coordinate plane, again plot the functions $y = \sin x + 5$ and $y = \cos x + 3$. Shade the region under the curve $y = \cos x + 3$ over the interval [2, 5] and label it A2.
        - What does A1 represent?
        - What does A2 represent?
        - What do you get if you subtract A2 from A1?
    - Based on your observations, what can you conclude about finding the area of the region between two curves $y = f(x)$ and $y = g(x)$ over the interval [a,b] given that $f(c) \geq g(x)$ for all x in $[a, b]$?

Volume (Disk Method) - Determine the volume of a solid created by a continuous function that is rotated around a vertical or horizontal line (disk method).

- *explore graphic interpretation of finding the volume of multiple cross-sections*
- *use technology to examine the shapes of solids created by rotated a curve around an axis*
- *use the definition of volume to find the volume of a curve that is rotated around the x-axis*
- *use the definition of volume to find the volume of a curve that is rotated around the y-axis*

Volume (Cylindrical Shells Method) - Determine the volume of a solid by using the method of cylindrical shells.

- *identify the graphs of volumes that are easier to calculate using the method of cylindrical shells*
- *use technology and graphing to visualize the shape created by rotating*
- *visualize the cylindrical shells that will be used make up the volume of the solid*
- *perform integration to find volume*

Work - Use the definite integral to calculate work.

- *examine the relationships between work, force, and distance*
- *relate the definite integral to work*
- *calculate work*

Mean Value Theorem for Integrals - Use the mean value theorem for integrals to find the average value of a function.

- *illustrate the average value of a function graphically*
- *compute the average value of a function*

Unit 3 – Inversion Functions (4-5 Weeks, 11 Lessons)

Inverse Functions - Calculate the inverse of a function.

- *explore the concept of one-to-one functions*
- *graph a function and its inverse*
- *find the inverse function of a one-to-one function*

Differentiating Exponential Functions - Differentiate exponential functions.

- *review the properties of exponential functions*
- *explore the graphs of exponential functions to determine where the function is continuous*
- *find the derivative of the exponential function using the definition of the derivative*
- *find the derivative of exponential functions that involve the product rule, quotient rule, and chain rule, etc.*

Integrating Exponential Functions - Evaluate integrals that include exponential functions.

- *evaluate definite and indefinite integrals that include the exponential function (bases a and e)*

Differentiating Logarithmic Functions - Differentiate logarithmic functions.

- *review the properties of logarithmic functions*
- *explore the graphs of logarithmic functions to determine where the function is continuous*
- *find the derivative of logarithmic functions*
- *find the derivative of logarithmic functions that involve the product rule, quotient rule, and chain rule, etc.*
- *use logarithmic differentiation to find the derivative of a complicated function that includes products, quotients, or powers.*

The Logarithm as an Integral - Evaluate integrals that include logarithmic functions.

- *evaluate definite and indefinite integrals that result in logarithmic functions (base e)*

Exponential Growth and Decay - Complete problems that involve exponential growth and decay.

- *use the formulas to solve problems using laws of natural growth and decay*

L'Hôpital's Rule - Explore indeterminate forms and l'Hôpital's rule.

- *use l'Hôpital's rule to evaluate the limit of function*

Differentiating Inverse Trigonometric Functions - Differentiate inverse trigonometric functions.

- *explore the graphs of trigonometric functions and their inverses*
- *perform differentiation on inverse trigonometric functions*

Integration Formulas for Inverse Trigonometric Functions - Evaluate integrals of inverse trigonometric functions.

- *evaluate definite and indefinite integrals that require inverse trigonometric functions*

Differentiating Hyperbolic Functions - Differentiate hyperbolic functions.

- *explore the graphs of hyperbolic functions*
- *perform differentiation on hyperbolic functions*

Integrating Hyperbolic Functions - Evaluate integrals of hyperbolic functions.

- *explore the graphs of exponential functions and hyperbolic functions*
- *use formulas and substitution to evaluate the integrals of hyperbolic functions*

Unit 4 – Techniques of Integration (3-4 Weeks, 8 Lessons

Integration by Parts - Evaluate integrals by using integration by parts.

- *identify integrals that cannot be completed using previously learned methods*
- *find an indefinite integral using integration by parts*
- *find a definite integral using integration by parts*

Strategies for Evaluating Complex Trigonometric Functions - Evaluate integrals that include powers of trigonometric functions.

- *use trigonometric identities to form strategies for evaluating trigonometric functions involving powers*
- *evaluate indefinite and definite integrals of trigonometric functions*

Trigonometric Substitution - Evaluate integrals that can be solved by trigonometric substitution.

- *identify integrals that can be handled using trigonometric substitution*
- *use the trigonometric functions and the parts of a triangle to determine valid substitutions*
- *perform integration using trigonometric substitution*

Integration by Partial Fractions and Long Division - Evaluate integrals of rational functions.

- *use long division of algebraic statements to rewrite a rational function*
- *use partial fractions to rewrite rational functions*
- *evaluate indefinite and definite integrals of rational functions*

Using Integration Tables - Evaluate integrals using the tables of integration formulas.

- *describe the categories of integrals included in tables*
- *use tables and substitution to evaluate integrals*

Strategies for Integrating Functions - Strategize methods for integration.

- *examine all methods for evaluating integrals*
- *determine strategy and evaluate integrals using any method*
- *explore technology that can be used to evaluate integrals*

Methods for Approximating an Integral - Calculate approximations of integrals.

- *explore the midpoint rule graphically*
- *calculate the area using the midpoint rule*
- *explore the trapezoidal rule graphically*
- *calculate the area using the trapezoidal rule*
- *explore Simpson's rule graphically*
- *calculate the area using Simpson's rule*
- *compare the results of calculating area using the trapezoidal rule and Simpson's rule.*

Improper Integrals - Evaluate improper integrals.

- *evaluate integrals that converge and diverge*
- *evaluate integrals that are discontinuous*

Unit 5 – Further Applications of Integration (1-2 Weeks, 5 Lessons)

Arc Length - Calculate the arc length of a continuous curve.

- *estimate the length of a curve graphically by dividing the curve into 2, 4, 6, etc. segments*
- *use the arc length formula to calculate the length of a curve using either dy/dx or dx/dy*
- *apply the use of arc length in practical setting*

Surface Area - Calculate the surface area of a revolution.

- *use technology and graphing to visualize the surface area of a solid*
- *compare techniques of finding basic solids to solids created from rotating an equation about a line*
- *calculate the surface area of a revolution by using either dy/dx or dx/dy*
- *apply the use of surface area in a practical setting*

Calculus in Physics, Engineering, and Biology - Explore applications to physics, biology and engineering.

- *create and evaluate integrals that represent volumes and surface areas which may include pressure and center of mass*
- *create and evaluate integrals that represent blood flow, cardiac output, etc.*

Calculus in Economics - Explore applications to economics.

- *create and evaluate integrals that represent consumer surplus*

Calculus in Probability - Explore applications to probability.

- *create and evaluate integrals that represent are below the curve which represent probabilities*
- *use the function for the normal distribution to evaluate areas below the curve (compare to tables)*